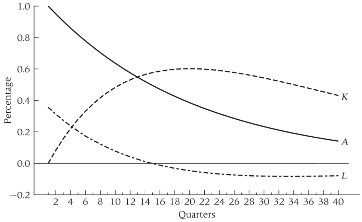

Advanced Macroeconomics

Section 4 - Fluctuations (I): RBC theory

Daniele Girardi
King's College London

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Influential stylized facts

1. Significant short-run fluctuations (‘cycles’) around growth path, but no apparent regularity
2. Investment much more pro-cyclical than consumption
3. Productivity is pro-cyclical
4. Real wages don’t react much to short-run fluctuations (slightly pro-cyclical)

RBC theory: overview

- Ramsey economy hit by temporary random shocks
- shocks are ‘real’, and persistent
  - they (temporarily) affect the parameters which determine the optimal growth path of the economy.
  - So they directly affect real (not nominal) magnitudes.
  - Typically: productivity parameter.
- they propagate through the mechanics of the model
- short-run fluctuations as optimal responses to temporary shocks
- policy implication: stabilization policies (monetary & fiscal) are not needed or helpful.

RBC theory: overview

- A Ramsey model with two modifications:
  1. Temporary random disturbances to the key parameters;
  2. Endogenous labor supply.

RBC theory: overview

- A Ramsey model with two modifications:
  1. Temporary random disturbances to the key parameters;
  2. Endogenous labor supply.
- Methodologically, it won the day;
- but (almost) no one considers it even remotely realistic.

The plan

1. Outline the baseline RBC model;
2. derive two utility-maximization conditions that will come in handy for solving the model;
3. solve the model in the special case of no G and only circulating K;
4. describe the main features of the solution to the general case;
5. discussion: plausibility, extensions and shortcomings;

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Assumptions about production

- many identical price-taking firms;
- many identical infinitely-lived households;
- Cobb-Douglas production:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1$$

- Evolution of K:

$$K_{t+1} = K_t + I_t - \delta K_t$$

- Factor remunerations:

$$w_t = MPL_t \qquad r_t = MPK_t - \delta$$

Behavioral assumptions

- Representative household maximizes expected value of

$$U = \sum_{t=0}^{\infty} e^{-\rho t} u(c_t, 1-\ell_t)\frac{N_t}{H}$$

- Instantaneous utility:

$$u_t = \ln c_t + b\ln(1-\ell_t), \quad b > 0$$

- H fixed but (exogenous) population growth:

$$\ln N_t = \bar{N} + nt, \quad n < \rho$$

Technological change and government

- *Technological progress:*

$$\ln A_t = \bar{A} + gt + \tilde{A}_t$$

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A,t}, \qquad -1 < \rho_A < 1$$

  - $\epsilon_{A,t}$ = uncorrelated zero-mean disturbance (*white-noise*)
  - ρ_A makes the shocks persistent (by assumption).

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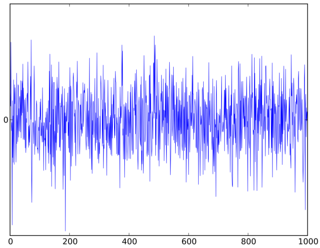
- *Government purchases:*

$$\ln G_t = \bar{G} + (n+g)t + \tilde{G}_t$$

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}, \quad -1 < \rho_G < 1$$

- (same logic)

Example of white-noise ϵ_t term:



- Value in each period is a random draw with mean zero, independent of previous periods (uncorrelated);
- Persistence is added through the ρ terms;

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(1) Euler Equation

- 'Intuitive derivation'
- Optimization requires that a marginal reallocation of C from t to $t+1$ does not affect expected utility
- Δc decrease in consumption per head in t
  $\Rightarrow e^{-n}(1+r_{t+1})\Delta c$ increase in consumption per head in $t+1$.
- Equating utility cost and (expected) utility benefit:

$$\left(e^{-\rho t}\frac{N_t}{H}\frac{1}{c_t}\right)\Delta c = E_t\left[\left(e^{-\rho(t+1)}\frac{N_{t+1}}{H}\frac{1}{c_{t+1}}\right)\frac{1+r_{t+1}}{e^n}\Delta c\right]$$

- Rewrite as

$$\frac{1}{c_t} = e^{-\rho}E_t\left[\frac{1+r_{t+1}}{c_{t+1}}\right]$$

(2) Optimal labor supply

- ‘Intuitive derivation’ again
- Optimization requires that a marginal increase in ℓ_t, using resulting income to raise c_t, does not affect expected utility
  - $$\left(e^{-\rho t}\frac{N_t}{H}\frac{b}{1-\ell_t}\right)\Delta\ell = \left(e^{-\rho t}\frac{N_t}{H}\frac{1}{c_t}\right)w_t\Delta\ell$$
  - *marginal disutility of* ℓ_t = $w_t \times$ *marginal utility of* c_t
- Rewrite as

$$\frac{c_t}{1-\ell_t} = \frac{w_t}{b}$$

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Solving (a special case of) the model

- Given stochastic shocks, household can't choose deterministic future paths for C and L at time 0, but needs to 'recalculate' in each period based on shocks realization
  - (like Google Maps when there are changes in traffic).

- Unfortunately, baseline RBC model cannot be solved analytically (mix of linear and log-linear equations).

- Special case that can be solved analytically: only circulating K ($\delta = 1$) and no government

$$K_{t+1} = I_t = Y_t - C_t$$

$$1 + r_t = \alpha \left(\frac{A_t L_t}{K_t} \right)^{1-\alpha}$$

Solving the (special case of) the RBC model

- as usual, we are after the intertemporal equilibrium;
- two state variables: K and A;
- two endogenous variables: C and L;
- conjecture: s and ℓ constant in equilibrium;

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- as usual, we are after the intertemporal equilibrium;
- two state variables: K and A;
- two endogenous variables: C and L;
- conjecture: s and ℓ constant in equilibrium;
- Strategy:
  1. use equilibrium conditions for household behavior to find $\hat{s}$ and $\hat{\ell}$;
  2. Study equilibrium behavior of aggregates;

Step 1: use Euler Equation to find $\hat{s}$.

- Rewrite EE in terms of s using $c_t = (1-s_t)\frac{Y_t}{N_t}$, and take logs:

$$-\ln\left[(1-s_t)\frac{Y_t}{N_t}\right] = -\rho + \ln E_t\left[\frac{1+r_{t+1}}{(1-s_{t+1})Y_{t+1}/N_{t+1}}\right]$$

- Setting $s_t = s_{t+1} = \hat{s}$ and solving for $\hat{s}$, this implies:

$$\hat{s} = \alpha e^{n-\rho}$$

- confirms the conjecture that there is a solution with constant s.

Step 2: use optimal labor supply condition to find $\hat{\ell}$.

- Rewrite labor supply condition in terms of $\hat{s}$ using $c_t = (1-\hat{s})\frac{Y_t}{N_t}$, and take logs:

$$\ln\left[(1-\hat{s})\frac{Y_t}{N_t}\right] - \ln(1-\ell_t) = \ln w_t - \ln b$$

- Using $w_t = MPL = (1-\alpha)Y_t/(\ell_t N_t)$ and solving for ℓ, we get

$$\ell_t = \frac{1-\alpha}{(1-\alpha)+b(1-\hat{s})} = \hat{\ell}$$

- confirms that there is a solution with constant ℓ as well as constant s.

RBC special case: implications for output dynamics

- From production function (taking logs):

$$\ln Y_t = \alpha \ln K_t + (1-\alpha)(\ln A_t + \ln L_t)$$

- Using definitions and assumptions on K, L, N and A, and the fact that $s = \hat{s}$ and $\ell = \hat{\ell}$, and after lots of boring (but easy) algebra, you get:

$$\tilde{Y}_t = (\alpha + \rho_A)\tilde{Y}_{t-1} - (\alpha\rho_A)\tilde{Y}_{t-2} + (1-\alpha)\epsilon_{A,t}$$

  - $\tilde{Y}_t$ = deviation of $\ln Y_t$ from its balanced growth-path.

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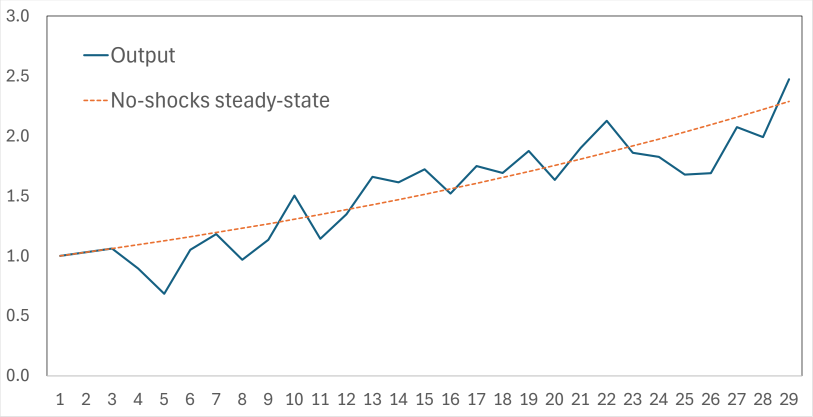
- Bottom line: Transitory hump-shaped fluctuations around balanced growth path

- fixed s and ℓ $\rightarrow$ wages, consumption and investment are all highly pro-cyclical;

A simulated example

- $\alpha = 0.3$, $\rho_A = 0.25$, $SD(\epsilon_{A,t}) = 1$, $Y_0 = 1$, $g = 0.02$, $n = 0.01$.



The general case

- Cannot be solved analytically;
- Log-linear approximation to the model
  - log-linearize around the steady-state
  - 1st order Taylor approximation in the logs of key variables
  - can be solved analytically
- Method of undetermined coefficients:
  1. conjecture the general functional form of the (log-linearized) solution;
  2. use the equations of the model to pin down coefficients' values.

Method of undetermined coefficients

- Define $\tilde{X}_t = \ln X_t - \ln X^*$
- Log-linearization around steady-state:

$$\tilde{C}_t \approx a_{CK}\tilde{K}_t + a_{CA}\tilde{A}_t + a_{CG}\tilde{G}_t$$

$$\tilde{L}_t \approx a_{LK}\tilde{K}_t + a_{LA}\tilde{A}_t + a_{LG}\tilde{G}_t$$

- Then use the two equilibrium conditions for household behavior to pin down the a's.

Optimal labor supply condition

- Labor-supply equilibrium condition:

$$\frac{c_t}{(1-\ell_t)} = \frac{w_t}{b}$$

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- using the fact that $w = (1-\alpha)[K_t/(A_t L_t)]^{\alpha} A_t$ and taking logs

$$\ln c_t - \ln(1-\ell_t) = \ln\left(\frac{1-\alpha}{b}\right) + (1-\alpha)\ln A_t + \alpha ln K_t - \alpha \ln L_t$$

- Take log-linear approximation around balanced growth path
    - it means that for both sides, we compute the (approximate) deviation from its steady state value.

$$\tilde{C}_t + \frac{\ell^\star}{1-\ell^\star}\tilde{L}_t = (1-\alpha)\tilde{A}_t + \alpha\tilde{K}_t - \alpha\tilde{L}_t$$

(see handout on Keats for the detail of this log-linearization)

Baseline RBC model

- Log-linear approximation of the optimal ℓ condition

$$\tilde{C}_t + \frac{\ell^\star}{1-\ell^\star}\tilde{L}_t = (1-\alpha)\tilde{A}_t + \alpha\tilde{K}_t - \alpha\tilde{L}_t$$

- Substitute the general-form solutions for $\tilde{C}$ and $\tilde{L}$ into this:

$$a_{CK}\tilde{K}_t + a_{CA}\tilde{A}_t + a_{CG}\tilde{G}_t + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha\right)(a_{LK}\tilde{K}_t + a_{LA}\tilde{A}_t + a_{LG}\tilde{G}_t) =$$

$$= \alpha\tilde{K}_t + (1-\alpha)\tilde{A}_t$$

- Log-linear approximation of the optimal ℓ condition

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$$= \alpha\tilde{K}_t + (1-\alpha)\tilde{A}_t$$

- ... and finally equate the coefficients on $\tilde{K}$, $\tilde{A}$ and $\tilde{G}$ on both sides

$$a_{CK} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha\right)a_{LK} = \alpha; \quad a_{CA} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha\right)a_{LA} = 1-\alpha$$

$$a_{CG} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha\right)a_{LG} = 0$$

Coefficients restrictions from optimal labor supply condition:

$$a_{CK} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha \right) a_{LK} = \alpha \tag{1}$$

$$a_{CA} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha \right) a_{LA} = 1-\alpha \tag{2}$$

$$a_{CG} + \left(\frac{\ell^\star}{1-\ell^\star} + \alpha \right) a_{LG} = 0 \tag{3}$$

1. implies that either C or L (or both) must respond positively to $K\uparrow$;
2. implies that either C or L (or both) must respond positively to $A\uparrow$;
3. implies that the responses of C and L to $G\uparrow$ must have opposite sign;

Euler Equation

- Using the EE, we can add further restrictions.
- Pin down the a's as a function of the model parameters $(\alpha, g, n, \delta, \rho_A, \rho_G, \bar{G}, \rho, b)$;
- following D. Romer's wisdom, we skip this (very complicated) derivation;
- 'Impulse response functions': simulate effect of exogenous A and G shocks on the endogenous variables;
- If you feel going through derivation details would be helpful for you (eg, if you plan to work on macro modeling), you can study Campbell (1994) or similar;

Impulse response functions from the baseline RBC model

Assumptions about parameter values:

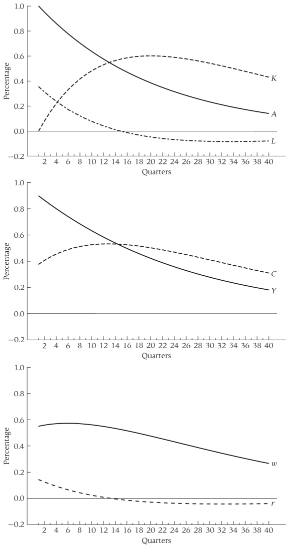
- A period is a quarter;
- $\alpha = \frac{1}{3}$
- $g = 0.5\%$
- $n = 0.25\%$
- $\delta = 2.5\%$
- $\rho_A = \rho_G = 0.95$;
- $\bar{G}$ such that $G/Y = 0.2$
- ρ such that $r^\star = 1.5\%$;
- b such that $\ell^\star = \frac{1}{3}$.

$\rightarrow$

Resulting coefficients:

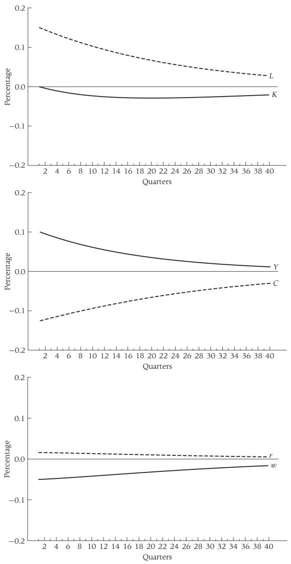
- $a_{LA} \approx 0.35$
- $a_{LK} \approx -0.31$
- $a_{CA} \approx 0.38$
- $a_{CK} \approx 0.59$
- $b_{KA} \approx 0.08$
- $b_{KK} \approx 0.95$

Effect of a positive technology shock



- *A* shock itself is temporary but persistent (by assumption);
- temporarily raises *A*, *K* and *L*;
- *intertemporal substitution* (temporarily higher MPK & MPL) makes L and I increase.
- but it is partly offset by a *wealth effect* (higher PV of lifetime wealth) with opposite effect: $C\uparrow$; $L\downarrow$;
- wages and interest rate rise, but not by much
  - increase in *L* and *K* partly offset the positive effect of the technology shock on MPL and MPK;

Effect of a gov't purchases shock



- *G* shock itself is transitory but persistent (by assumption);
- Less output available for *C* and *I*;
- *Negative wealth effect:* *C* ↓ and *L* ↑;
- *Intertemporal substitution:* *K* ↓ to smooth-out consumption, which mitigates the decrease in *C*;
- Output rises (very slightly) because of higher labor supply, which also makes wages slightly decrease;

Calibration of RBC models

1. Choose specific functional forms and values for all model parameters
2. Pick some variances and covariances of macro variables, and compare the observed ones with the ones generated by simulations of the model

TABLE 5.4 A calibrated real-business-cycle model versus actual data

	U.S. data	Baseline real-business-cycle model
σ_Y	1.92	1.30
σ_C/σ_Y	0.45	0.31
σ_I/σ_Y	2.78	3.15
σ_L/σ_Y	0.96	0.49
$\mathrm{Corr}(L, Y/L)$	-0.14	0.93

Source: Hansen and Wright (1992).

Main extensions to the baseline RBC model

Indivisible labor:

- hours worked are a discrete variable;
- raises σ_Y and σ_L/σ_Y

Multiple sectors

- effect of sector-specific technological shocks (hypothesis that sector-specific shocks can significantly explain aggregate fluctuations).

Distortionary taxation

- Distortionary taxes ($T_t = \tau Y_t$) to finance government purchases;
- they will make people work less (intertemporal substitution), making fiscal expansions *contractionary*;

Why (almost) no one believes RBC theory (1/2)

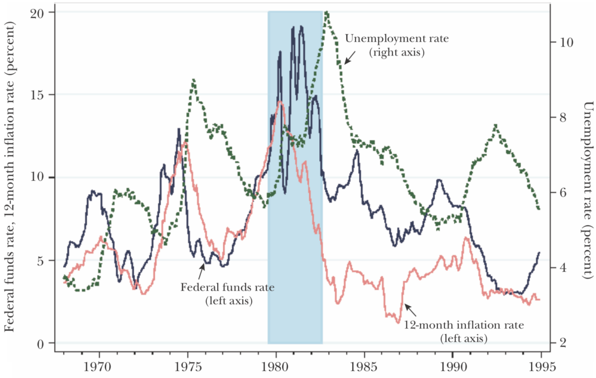
- No involuntary unemployment?
- What are the ‘productivity shocks’? Why dont’t we read about them in the newspaper?
  - “*[RBC models] attribute fluctuations in aggregate variables to imaginary causal forces that are not influenced by the action that any person takes.*”(P. Romer, 2016)
  - Consumption is micro-founded but productivity is not!
  - Seems more likely that short-run fluctuations in productivity reflect changes in utilization rates due to aggregate demand shocks.

Why (almost) no one believes RBC theory (2/2)

- Deviations from the perfect-and-complete markets model are so pervasive that it seems incredible that they dont't have any substantial effect on the macroeconomy;
- A truckload of evidence that monetary policy can affect real variables;

Monetary non-neutrality
Exhibit 1: the 'Volcker recession' (1979-1982)

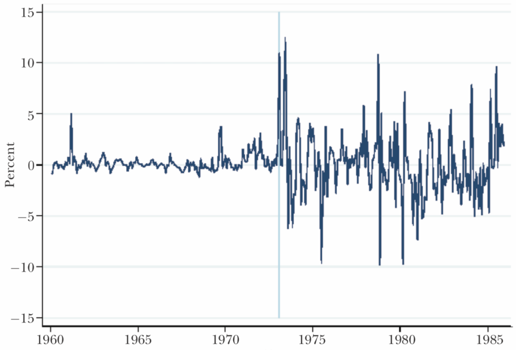
Figure 2
Federal Funds Rate, Inflation, and Unemployment from 1965 to 1995



Note: The figure plots the federal funds rate (dark solid line, left axis), the 12-month inflation rate (light solid line, left axis), and the unemployment rate (dashed line, right axis). The Volcker disinflation period is the shaded bar (August 1979 to August 1982).

Monetary non-neutrality
Exhibit 2: The end of the Bretton Woods system

Figure 3
Monthly Change in the US–German Real Exchange Rate



Note: The figure plots the monthly change in the US–German real exchange rate from 1960 to 1990. The vertical line marks February 1973, when the Bretton Woods system of fixed exchange rates collapsed.