



## Labor market models

- ▶ Key stylized facts:
  - persistent and substantial involuntary unemployment
  - limited pro-cyclicality of wages
  - strong pro-cyclicality of employment
  - at odds with a plain neoclassical demand-supply model
- ▶ *Efficiency-wages* [Bowles-Stiglitz-Shapiro]
- ▶ *Search-and-matching* [Diamond-Mortensen-Pissarides]
- ▶ *Monopsony* [Manning 2003, Dube et al., 2018, Azar et al. 2019, ...]

## Employment contracts are *incomplete* contracts

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- ▶ nor how much *effort* the worker must exert on the job.
- ▶ Effort is hard to observe, measure and prove in court.

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- ▶ Effort is hard to observe, measure and prove in court.
- ▶ With an incomplete contract, power matters
- ▶ Employers use their position of power (the threat of the sack) to obtain effort from workers.
- ▶ But for employers to be in a position of power, they have to pay an *efficiency wage*, not a market-clearing wage.

### Equilibrium unemployment as a discipline device

- ▶ Unemployment emerges in equilibrium as a *discipline device*.
- ▶ Crucial for the functioning of the labor market: it makes it costly for workers to lose their job, thus inducing adequate work effort.
- ▶ *Wage curve*: the lower the unemployment rate, the higher the equilibrium wage.

## A very simplified efficiency-wage model

- ▶ Abstract from dynamics and focus on one single period.
- ▶ A representative firm hires a representative worker.
- ▶ Worker chooses how much effort to exert
- ▶ Firm can imperfectly observe the effort level of the worker.
- ▶ Firm chooses the wage to offer and a *termination schedule*.
- ▶ The termination schedule relates the probability of employment termination to its (imperfect) observation of the worker's effort.

## Assumptions

- ▶ Employer wants to maximise the effort-wage ratio:

$$\Pi = e/w \quad \text{with } 0 \leq e \leq 1$$

- ▶ Worker gets utility from income and disutility from effort:

$$u(y, e) = y - \frac{a}{1 - e} \quad \text{with } a > 0$$

- ▶ Worker income equals the wage  $w$  if not terminated.
- ▶ If terminated, worker gets unemployment benefit  $B/s$ , where  $s$  is the number of unemployed workers in the economy.
- ▶ Termination schedule determines probability of termination  $P_F$ :

$$P_F = P_F(e) = 1 - e$$

## The worker choice of effort level

- ▶ Worker expected utility

$$E(U) = [1 - P_F(e)]w + P_F(e) \left( \frac{B}{s} \right) - \frac{a}{1 - e}$$

- ▶ Defining the cost of job loss  $\hat{c} \equiv w - \frac{B}{s}$ ,

$$E(U) = [1 - P_F(e)]w + P_F(e) (w - \hat{c}) - \frac{a}{1 - e}$$

- ▶ Expected utility maximization implies:

$$e^* = 1 - \left( \frac{a}{\hat{c}} \right)^{\frac{1}{2}}$$



## The worker optimal effort function

$$e^* = 1 - \left( \frac{a}{\hat{c}} \right)^{\frac{1}{2}} \quad \text{with} \quad \hat{c} \equiv w - \frac{B}{s}$$

- ▶ Worker effort is an increasing function of the cost of job loss.
- ▶ Higher wage  $\rightarrow$  more effort.
- ▶ Higher generosity of unemployment benefits  $B \rightarrow$  less effort.
- ▶ Higher unemployment  $s \rightarrow$  more effort
- ▶ Higher disutility of effort  $a \rightarrow$  less effort.

## The firm choice of a wage offer

- ▶ Firm knows the worker optimal effort function, so maximizes

$$\Pi = e^*(w)/w$$

- ▶ So their optimal wage offer is

$$w^* = \frac{e^*(w)}{e_w^*(w)}$$

$$\circ \max_w \frac{e(w)}{w} \Rightarrow \frac{\partial \frac{e(w)}{w}}{\partial w} = 0 \Rightarrow w = \frac{e(w)}{e_w(w)}$$

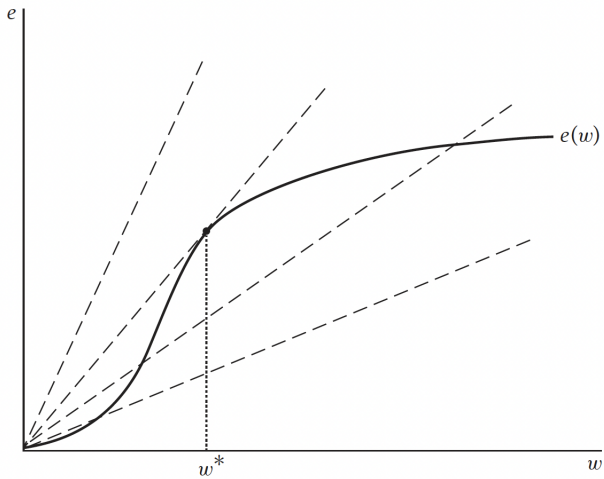
- ▶ This also implies that in equilibrium

$$\frac{e}{w} = e_w$$

*At the optimal wage offer, the slope of the iso-profit curve equals the slope of the worker optimal effort function (graph next slide).*

# Simplified efficiency-wage model

## Equilibrium wage and effort



## The wage curve

- ▶ An increase in unemployment  $s$  shifts the worker optimal effort function up (higher effort for each given wage).
- ▶ Therefore it leads to a lower equilibrium wage.
- ▶ Intuition: unemployment raises the cost of job loss, so a lower wage is necessary to induce adequate effort.
- ▶ *Wage curve*: The wage is a negative function of unemployment  $s$ .
- ▶ This provides a micro-foundation for the wage-setting curve in our simplified New Keynesian model!

### Takeaways

- ▶ The model provides a possible explanation for unemployment
  - Efficiency wage  $>$  market-clearing wage.
- ▶ Moreover, the equilibrium outcome is inefficient for firm and worker
  - It can be shown that it would be possible to increase both the worker utility and the firm profit by choosing a higher wage and higher effort.
  - Coordination failure.
- ▶ *Power matters*: Firm uses the threat of termination to discipline workers into exerting effort on the job.

# Search-and-matching

- ▶ No Walrasian centralized market clearing.
- ▶ Workers & firms meet in decentralized one-on-one matches.
- ▶ Costly and time-consuming search process produces 'frictional' unemployment.
- ▶ Diamond-Mortensen-Pissarides model (2010 Nobel Prize).

## Assumptions about the economy

- ▶ Continuum of workers of mass 1.
- ▶ Firms open vacancies and then search for workers.
- ▶ Maintaining a job (filled or unfilled) costs  $c$  to the firm.
- ▶ Firm's payoff per period from a job:
  - $y - w(t) - c$  if filled.
  - $-c$  if unfilled.
- ▶ Worker's payoff per period:
  - $w$  if employed.
  - $b$  if unemployed.
- ▶  $y > b + c$ , so there is always positive surplus from filling a job.

## Assumptions about job matching

- ▶ At each point in time  $M(t)$  job matches occur.
- ▶ Matching function:

$$M(t) = M[U(t), V(t)], \quad M_U > 0; \quad M_V > 0$$

- ▶ Jobs end at an exogenous rate  $\lambda$ .
- ▶ Employment change:

$$\dot{E}(t) = M(U(t), V(t)) - \lambda E(t)$$

- ▶ Share  $\phi$  of surplus from filling a vacancy goes to the worker (bargaining power).



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- ▶ Job-finding rate:

$$a(t) = \frac{M(t)}{U(t)} = m[\theta(t)]$$

- ▶ Vacancy-filling rate:

$$\alpha(t) = \frac{M(t)}{V(t)} = \frac{m[\theta(t)]}{\theta(t)}$$

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- ▶ Specifically, Cobb-Douglas matching function:

$$M(U, V) = kU^{1-\gamma}V^\gamma; \quad m(\theta) = k\theta^\gamma$$

## Solving the model

- ▶ We are after the *inter-temporal equilibrium* (steady state); we'll ignore disequilibrium dynamics & stability issues;
- ▶ **Strategy:**
  1. Figure out the value (= expected lifetime utility) of each state for each agent
    - $V_E, V_U, V_F, V_V$
  2. Impose intertemporal equilibrium conditions (constant  $V$ 's,  $E$ ,  $a$ ,  $\alpha$ ).
  3. Find  $V_V$  as a function of  $E$  and exogenous parameters;
  4. Impose equilibrium condition  $V_V = 0$  to determine the equilibrium values of  $E$ ,  $a$  and  $\alpha$ .

## 1 - Value of each possible state

- ▶ Value of being employed:

$$rV_E(t) = w(t) - \lambda[V_E(t) - V_U(t)] + \dot{V}_E(t)$$

- ▶ Value of being unemployed:

$$rV_U(t) = b + a(t)[V_E(t) - V_U(t)] + \dot{V}_U(t)$$

- ▶ Value of a filled job:

$$rV_F(t) = [y - w(t) - c] - \lambda[V_F(t) - V_V(t)] + \dot{V}_F(t)$$

- ▶ Value of a vacancy:

$$rV_V(t) = -c + \alpha(t)[V_F(t) - V_V(t)] + \dot{V}_V(t)$$

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## 2 - Steady-state conditions

- ▶  $\dot{E} = \dot{\alpha} = \dot{a} = \dot{V}_E = \dot{V}_U = \dot{V}_F = \dot{V}_V = 0;$

Step 3 - Find  $V_V$  as a function of  $E$ 

- ▶ Model equations + steady-state conditions imply (after some algebra)

$$rV_V = -c + \frac{[(1-\phi)\alpha(E)](y-b)}{\phi a(E) + (1-\phi)\alpha(E) + \lambda + r}, \quad a_E > 0, \alpha_E < 0 \quad \Rightarrow \quad \frac{\partial V_V}{\partial E} < 0$$



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### Step 4 - Free-entry condition pins down equilibrium $E$

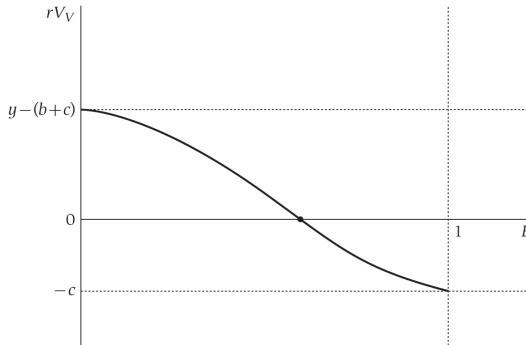
- ▶ Free-entry implies  $V_V = 0$

$$rV_V = -c + \frac{[(1-\phi)\alpha(E)](y-b)}{\phi a(E) + (1-\phi)\alpha(E) + \lambda + r} = 0$$

- ▶ This implicitly defines the equilibrium values of  $E$ ,  $a$  and  $\alpha$ .

## The equilibrium employment rate

- ▶ The equilibrium unemployment rate is implicitly defined by  $\frac{\partial V_V}{\partial E} < 0$  &  $V_V = 0$



## Takeaways

- ▶ Equilibrium unemployment could be just 'frictional'...
  - ▶ (...**but** evidence on long-term unemployment suggests otherwise; moreover, unemployment could seem frictional for the individual worker, while not being so on aggregate).
- ▶ cyclical increase in profitability of a filled job ( $y$  up, no change in  $c$  and  $b$ ) brings to large wage increase and modest increase in employment and vacancies
  - ▶ no wage rigidity!
  - ▶ (increase in job-finding rate pushes wages up, reducing incentive to create new vacancies);
- ▶ decentralized equilibrium is generally not efficient
  - ▶ see stylized example in the book.