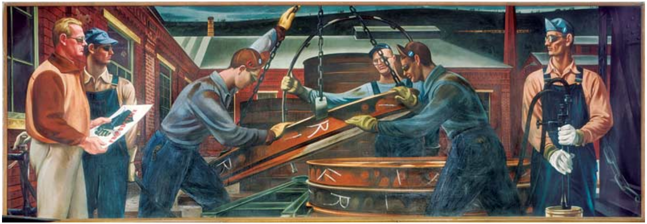


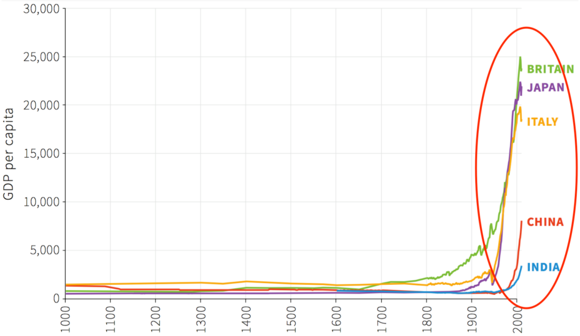
Advanced Macroeconomics

Section 2 - Growth (I): The mechanics of capital accumulation and growth

Daniele Girardi
King's College London

AY 2023-24, Semester I

The hockey stick of history



Section 2: Growth (I)

The Plan

1. Harrod-Domar
2. Solow
3. Ramsey-Cass-Koopmans
4. Diamond’s overlapping-generations (OLG)

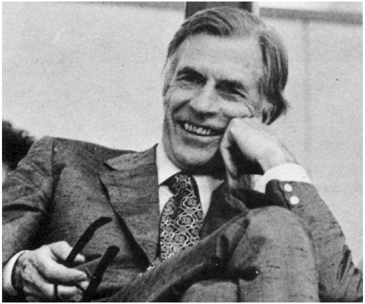
Key idea: Intertemporal equilibrium

- Static analysis: equilibrium condition –> equilibrium relations.
  - I=S
  - supply=demand
  - MRS=MRT
  - ...
- Growth theory is *dynamic*: intertemporal equilibria.

Main concepts:

- Intertemporal equilibrium
- Steady state
- Dynamic stability

The Harrod-Domar model



There's an old joke. Two elderly women are at a Catskill restaurant. One of them says, 'Boy, the food at this place is just terrible.' The other one says, 'Yeah I know. And such small portions.'

(Woody Allen, 'Annie Hall')

The Harrod-Domar model

- ‘Grandfather’ of modern growth theory.
- Premise 1: aggregate investment has a dual effect
  1. multiplier effect (demand side)
  2. capacity-creating effect (supply side)
- Premise 2: investment depends on output (accelerator)

The Harrod-Domar model

- ‘Grandfather’ of modern growth theory.
- Premise 1: aggregate investment has a dual effect
  1. multiplier effect (demand side)
  2. capacity-creating effect (supply side)
- Premise 2: investment depends on output (accelerator)
- Main findings:
  - unique equilibrium path: $g_W = sa$ (*warranted rate*)
  - warranted rate does not guarantee full (nor stable) employment
  - instability: economy won't converge to g_W, except by a fluke

Harrod-Domar model

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Harrod-Domar model

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Actual investment rate:

$$I(t) = S(t) \quad \rightarrow g_K(t) = \frac{S(t)}{K(t)} = s\frac{Y(t)}{K(t)} = s\frac{Y^*(t)}{K(t)}\frac{Y(t)}{Y^*(t)} = sa[u(t)]$$

Harrod-Domar model

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Actual investment rate:

$$I(t) = S(t) \quad \rightarrow g_K(t) = \frac{S(t)}{K(t)} = s\frac{Y(t)}{K(t)} = s\frac{Y^*(t)}{K(t)}\frac{Y(t)}{Y^*(t)} = sa[u(t)]$$

Intertemporal equilibrium (‘warranted’ growth rate):

$$\dot{g}_K(t) = 0 \quad \rightarrow u = 1 \quad \rightarrow g_W = sa$$

The equilibrium ('warranted') rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

The equilibrium ('warranted') rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

- Dynamic instability:

$$g_K = u(sa) > g_W = sa \quad \Rightarrow \quad u > 1 \quad \Rightarrow \quad \dot{g}_K > 0$$
$$g_K = u(sa) < g_W = sa \quad \Rightarrow \quad u < 1 \quad \Rightarrow \quad \dot{g}_K < 0$$

The equilibrium ('warranted') rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

- Dynamic instability:

$$g_K = u(sa) > g_W = sa \quad \Rightarrow \quad u > 1 \quad \Rightarrow \quad \dot{g}_K > 0$$
$$g_K = u(sa) < g_W = sa \quad \Rightarrow \quad u < 1 \quad \Rightarrow \quad \dot{g}_K < 0$$

- More formally:

$$\dot{g}_K = \frac{\alpha}{sa}(g_K - g_W) \quad \text{with } \alpha > 0$$

$$\dot{g}_K = \frac{\alpha}{sa}(g_K - g_w) \quad \text{with } \alpha > 0$$